p 467

41, ~~37~~, ~~29~~, 35, ~~37~~, 33

$$\boxed{\int_1^\infty \frac{1}{x}\,dx \text{ diverges}}$$

(33) $\displaystyle\int_\pi^\infty \frac{2+\cos x}{x}\,dx$

$$\frac{2+\cos x}{x} \ge \frac{1}{x}$$

$$\frac{1}{x^p}$$

$$\ln|x| \Big|_1^\infty$$

$$= \ln|\infty| - \ln|1|$$

(39) $\displaystyle\int_0^\infty \frac{16\tan^{-1} v}{1+v^2}\,dv = \int_0^\infty 16u\,du$

$u = \tan^{-1} v$

$du = \frac{1}{1+v^2}\,dv \quad = 16\frac{u^2}{2} = 8u^2 \Big|_0^\infty$

$$= 8(\tan^{-1} v)^2 \Big|_0^\infty = 8(\tan^{-1}\infty)^2 - 8(\tan^{-1} 0)^2$$

$$= 8\cdot\left(\frac{\pi}{2}\right)^2 - 8(0)$$

$$= 8\cdot\frac{\pi^2}{4} = 2\pi^2$$

Recap: Taylor Series

Taylor Series generated by f at $x=a$:

$$f(a)+\frac{f'(a)}{1!}(x-a)^1+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\cdots$$

$$\cdots+\frac{f^{(n)}(a)}{n!}(x-a)^n+\cdots$$

Ex) Approximate $\ln(1+x)$ with a 4th order Taylor Polynomial at $x=0$.

$f(0)=\underline{0}$, $f'(0)=\underline{1}$ $f''(0)=\underline{-1}$, $f'''(0)=\underline{-2}$

$f^{(4)}(0)=\underline{-6}$

$$P_4(x)=0+\frac{1}{1!}x^1+\frac{-1}{2!}x^2+\frac{-2}{3!}x^3+\frac{-6}{4!}x^4$$

or ...

Use series that we "know" as building blocks to create a new series.

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots$$

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}-\cdots$$

1. Find $P_4(x)$ for $\cos 2x$.

$$\cos 2x\approx P_4(x)=1-\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}$$

2. $\sin x+\cos x=\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\right)+\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots\right)$

$$=1+x-\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}-\cdots$$

3. $x\cos x+1$

$$x\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots\right)+1$$

$$=1+x-\frac{x^3}{2!}+\frac{x^5}{4!}-\frac{x^7}{6!}+\cdots$$

Taylor's Theorem with Remainder

If f has derivatives of all orders in some interval containing a, then

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n(x)$$

remainder = error

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$ for some value c between x and a.

HW: p 492 #37-42 all
p 500 #1, 3, 6, 7, 10